Chapter 9

Geometry

WHAT YOU WILL LEARN

- Points, lines, planes, and angles
- Polygons, similar figures, and congruent figures
- Perimeter and area
- Pythagorean theorem
- Circles
- Volume

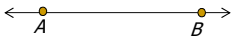
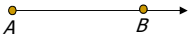
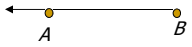
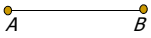
Section 9.1

Points, Lines, Planes, and Angles

Basic Terms

- A *point*, *line*, and *plane* are three basic terms in geometry that are NOT given a formal definition, yet we recognize them when we see them.
- A **line** is a set of points.
- Any two distinct points determine a unique line.
- Any point on a line separates the line into three parts: the point and two half lines.
- A **ray** is a half line including the endpoint.
- A **line segment** is part of a line between two points, including the endpoints.

Basic Terms

Description	Diagram	Symbol
Line *AB*		$\overleftrightarrow{AB}$
Ray *AB*		$\overrightarrow{AB}$
Ray BA		$\overrightarrow{BA}$
Line segment *AB*		$\overline{AB}$

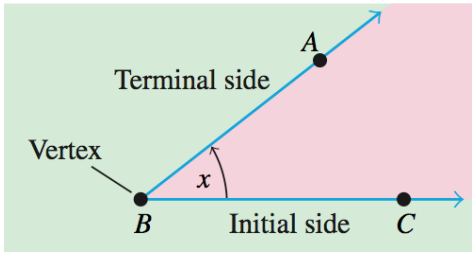
Plane

- We can think of a **plane** as a two-dimensional surface that extends infinitely in both directions.
- Any three points that are not on the same line (noncollinear points) determine a unique plane.
- A line in a plane divides the plane into three parts, the line and two half planes.
- Any line and a point not on the line determine a unique plane.
- The intersection of two distinct, non-parellel planes is a line.

Angles

- An **angle** is the union of two rays with a common endpoint; denoted $\angle$.
- The **vertex** is the point common to both rays.
- The **sides** are the rays that make the angle.
- There are several ways to name an angle:

$\angle ABC$, $\angle CBA$, $\angle B$



Angles

- The measure of an angle is the amount of rotation from its initial to its terminal side.
- Angles can be measured in *degrees, radians,* or *gradients.*
- Angles are classified by their degree measurement.
  - Right Angle is 90°
  - Acute Angle is less than 90°
  - Obtuse Angle is greater than 90° but less than 180°
  - Straight Angle is 180°

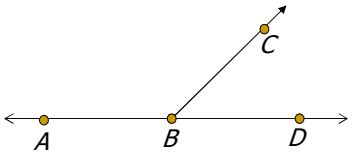
Types of Angles

- Adjacent Angles-angles that have a common vertex and a common side but no common interior points.
- Complementary Angles-two angles whose sum of their measures is 90 degrees.
- Supplementary Angles-two angles whose sum of their measures is 180 degrees.

Example

- If $\measuredangle ABC$ and $\measuredangle CBD$ are supplementary and the measure of ABC is 6 times larger than CBD, determine the measure of each angle.
- Let $x = m\measuredangle CBD$. Then:

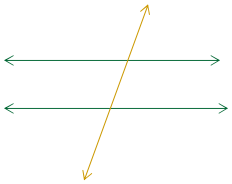
$$m\measuredangle ABC + m\measuredangle CBD = 180^\circ$$
$$6x + x = 180^\circ$$
$$7x = 180^\circ$$
$$x \approx 25.7^\circ$$



$$m\measuredangle ABC \approx 154.3^\circ$$
$$m\measuredangle CBD \approx 25.7^\circ$$

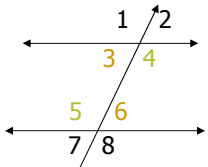
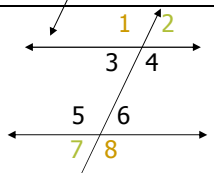
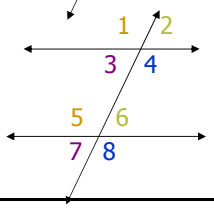
More definitions

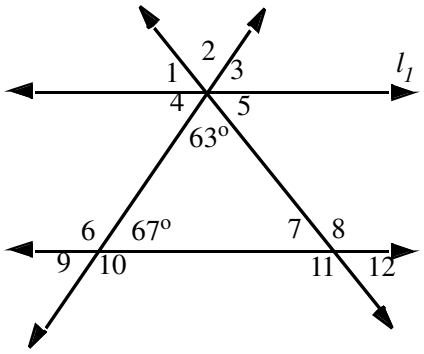
- **Vertical angles** are the nonadjacent angles formed by two intersecting straight lines.
- Vertical angles have the same measure.
- A line that intersects two different lines, at two different points is called a **transversal**.



- Special angles are given to the angles formed by a transversal crossing two parallel lines.

Special Names

Alternate interior angles	Interior angles on the opposite side of the transversal–have the same measure	
Alternate exterior angles	Exterior angles on the opposite sides of the transversal–have the same measure	
Corresponding angles	One interior and one exterior angle on the same side of the transversal–have the same measure	



$m\angle 1 =$ ____ $m\angle 7 =$ ____

$m\angle 2 =$ ____ $m\angle 8 =$ ____

$m\angle 3 =$ ____ $m\angle 9 =$ ____

$m\angle 4 =$ ____ $m\angle 10 =$ ____

$m\angle 5 =$ ____ $m\angle 11 =$ ____

$m\angle 6 =$ ____ $m\angle 12 =$ ____

Section 9.2

Polygons

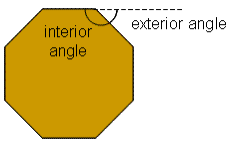
Polygons

- Polygons are named according to their number of sides.

Number of Sides	Name	Number of Sides	Name
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	20	Icosagon

Polygons (continued)

- The sum of the measures of the interior angles of an n-sided polygon is $(n - 2)180°$.
- Example: A certain brick paver is in the shape of a regular octagon. Determine the measure of an interior angle and the measure of one exterior angle.



Polygons (continued)

- Determine the sum of the interior angles.

$$\begin{aligned} S &= (n-2)180^\circ \\ &= (8-2)(180^\circ) \\ &= 6(180^\circ) \\ &= 1080^\circ \end{aligned}$$

- The measure of one interior angle is

$$\frac{1080^\circ}{8} = 135^\circ$$

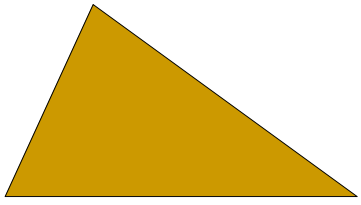
- The exterior angle is supplementary to the interior angle, so the measure of one exterior angle is

$180° – 135° = 45°$

Types of Triangles

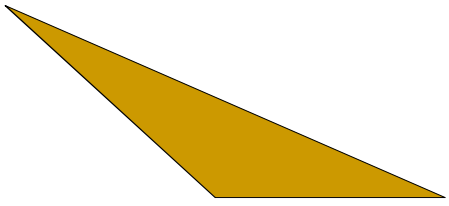
Acute Triangle

All angles are acute.



Obtuse Triangle

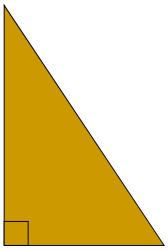
One angle is obtuse.



Types of Triangles (continued)

Right Triangle

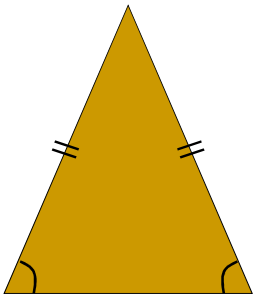
One angle is a right angle.



Isosceles Triangle

Two equal sides.

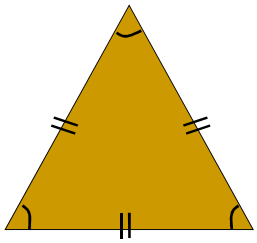
Two equal angles.



Types of Triangles (continued)

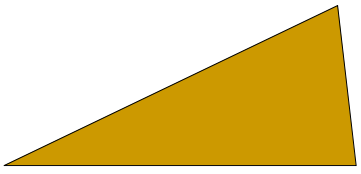
Equilateral Triangle

Three equal sides.
Three equal angles, 60º each.



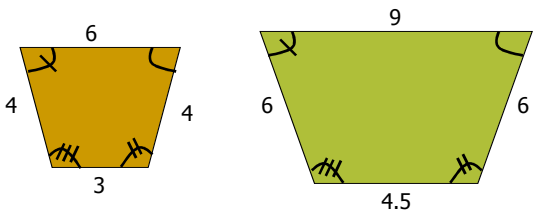
Scalene Triangle

No two sides are equal in length.



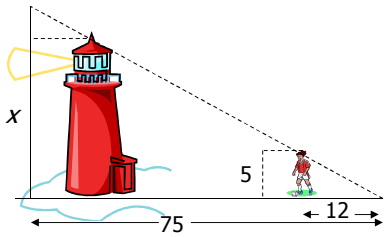
Similar Figures

- Two polygons are **similar** if their corresponding angles have the same measure and the lengths of their corresponding sides are in proportion.



Example

- Catherine Johnson wants to measure the height of a lighthouse. Catherine is 5 feet tall and determines that when her shadow is 12 feet long, the shadow of the lighthouse is 75 feet long. How tall is the lighthouse?



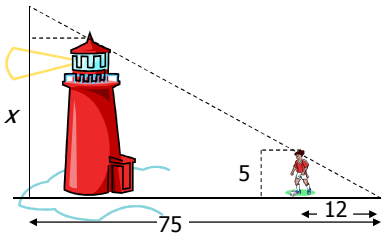
Example (continued)

$$\frac{\text{ht. lighthouse}}{\text{ht. Catherine}} = \frac{\text{length of lighthouse's shadow}}{\text{length of Catherine's shadow}}$$

$$\frac{x}{5} = \frac{75}{12}$$

$$12x = 375$$

$$x = 31.25$$



Therefore, the lighthouse is 31.25 feet tall.

Congruent Figures

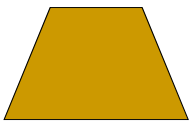
- If corresponding sides of two similar figures are the same length, the figures are **congruent**.
- Corresponding angles of congruent figures have the same measure.

Quadrilaterals

- Quadrilaterals are four-sided polygons, the sum of whose interior angles is 360°.
- Quadrilaterals may be classified according to their characteristics.

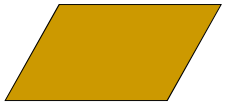
Classifications

- Trapezoid



Two sides are parallel.

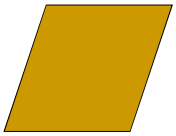
- Parallelogram



Both pairs of opposite sides are parallel. Both pairs of opposite sides are equal in length.

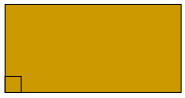
Classifications (continued)

- Rhombus



Both pairs of opposite sides are parallel. The four sides are equal in length.

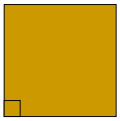
- Rectangle



Both pairs of opposite sides are parallel. Both pairs of opposite sides are equal in length. The angles are right angles.

Classifications (continued)

- Square



Both pairs of opposite sides are parallel. The four sides are equal in length. The angles are right angles.

GIVEN: Isosceles trapezoids ABCD and are similar figures.
Determine the length of side $\overline{A'B'}$